EXPERIMENTAL STUDY OF THE CENTRIPETAL AIR PUMPING EFFECT UNDER REDUCED PRESSURE

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UDC 532.58:532.526.75

Analyzed are the results of a study concerning the centripetal air pumping effect under reduced pressure.

In 1946 Weissenberg has noted that, if a vessel containing a high-viscosity polymer solution rotates about its axis while a stationary disk with a glass tube fitted into its center hole approaches the upper free surface of the liquid, the liquid will rise in the tube, i.e., at the center of the disk there will appear a positive excess pressure.

Earlier, Reiner [1] observed an effect in air which paralleled in its manifestations the Weissenberg effect. In [2] he ascribed this effect of centripetal pumping to the viscoelastic properties of air. These properties can be comprehensively interpreted with the aid of the mechanical model shown in Fig. 1 [2].

The model includes a Maxwell component consisting of a spring and a dashpot coupled in series. One end of this combination is fixed, the other end moves at a constant velocity. In addition, this model includes two springs aligned at some angle to the directions z and θ. The first spring is under compression, the second one is under tension. When point a is displaced downward at a constant velocity, then b moves downward by only an infinitesimal distance (represented by a deformation $D_{\theta z}$). This produces a deformation D_{zz} in the z-direction and a deformation $D_{\theta\theta}$ in the θ-direction. As a result, there appear a compressive stress S_{zz} and a tensile stress $S_{\theta\theta}$. The Reiner theory is based on the following rheological equation for a fluid:

$$\dot{D}_{lm(0)} = \dot{S}_{lm(0)}/2\mu + S_{lm(0)}/2\eta, \tag{1}$$

where the subscript 0 indicates a deviator.

The deformation is made up of two components: an elastic one (reversible)

$$D_{el} = S/2\mu \tag{2}$$

and a viscous one (irreversible)

$$D_{visc} = \frac{1}{2\eta} \int S d\tau.$$

On the basis of Eq. (1) Reiner tried to explain the centripetal pumping effect also known as the Reiner effect. Taylor and Saffman [3] explained this effect by the imprecision of the experiment. Starting with the fundamental Navier–Stokes equation, they have shown that this effect can occur as a result of a) a small error in the stator or rotor perpendicularity with respect to the axis of rotation, or b) axial vibrations of the rotor.

In these cases the total excess pressure would be

$$\text{a)} \quad P \approx e^2 \frac{\omega^2}{h^4} \cdot \frac{1}{P_0}, \tag{3}$$

Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 2, pp. 215-219, February, 1972. Original article submitted April 27, 1971.

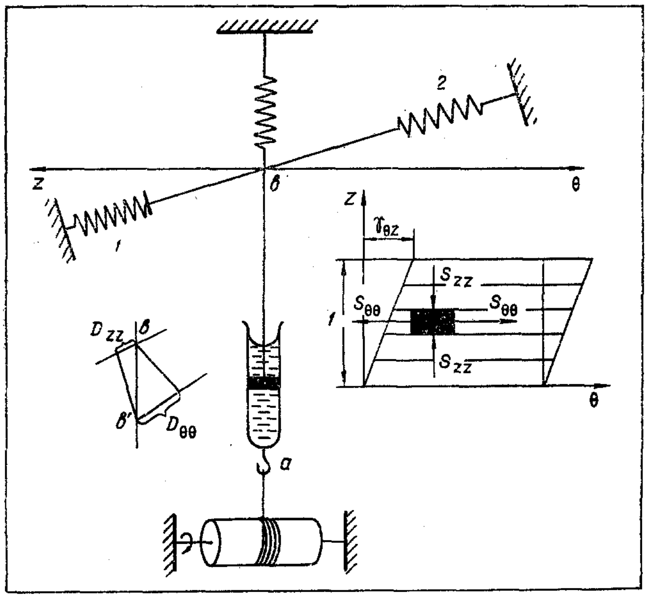

Fig. 1. Modified model of a Maxwell body: 1) compression spring; 2) tension spring; D_{zz}) contraction; $D_{\theta\theta}$) elongation.

$$\text{b)}\quad P \approx \alpha^2 \frac{\omega^2}{h^4} \cdot \frac{1}{P_0}, \tag{4}$$

where α and e are functions of the apparatus geometry and design but are independent of P_0.

It has been shown in [7] that when the gap is wide ($h \geq 50\ \mu$) and the pressure is low, however, a Reiner effect is observed which cannot be explained in the light of the Taylor–Saffman hypothesis (according to which this effect would be small).

According to the new theory of mechanics by Truesdell [4], the nonlinear equation of the viscous stress tensor S_{ij} includes a term which is proportional to the quantity $\eta \overset{0}{\dot{D}}_{ij}/P$ with $\overset{0}{\dot{D}}_{ij}$ denoting the shear rate tensor. This dimensionless quantity has been since called the Truesdell number Tr ($\mathrm{Tr} = \eta \cdot \overset{0}{\dot{D}}_{ij}/P$).

Considering a Maxwell distribution of molecule velocities, one can show that the Truesdell number is proportional to the Knudsen number and the Mach number:

$$\mathrm{Tr} = \frac{2}{3}\,\mathrm{Kn}\,\mathrm{Ma}. \tag{5}$$

It follows from the Truesdell theory that, when $\mathrm{Tr} \approx 1$, the centripetal effect will also occur in gases. It may be accompanied by a high Kn number or by a high Ma number.

A similar conclusion follows from the Predvoditelev theory [5, 6], according to which the transferable velocities of molecules vary along the path of their "free motion." Under this assumption the Navier–Stokes equation transforms into

$$(1-\beta)(\mathbf{V}\cdot\nabla)\mathbf{V} - \beta\mathbf{V}\,\mathrm{div}\,\mathbf{V} = F - \frac{1}{\rho}\nabla P + \frac{\eta}{\rho}\nabla^2 v + \frac{\eta_v}{\rho}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{V}, \tag{6}$$

where β denotes the continuity coefficient and Pd is the Predvoditelev number.

For a Maxwellian gas the Predvoditelev number is

$$\mathrm{Pd} \equiv |\beta| = \frac{3}{2}\,\mathrm{Kn}\,\mathrm{Ma}, \tag{7}$$

and, consequently,

$$\mathrm{Pd} = \frac{9}{4}\,\mathrm{Tr}, \tag{8}$$

i.e., the Predvoditelev number is proportional to the Truesdell number.

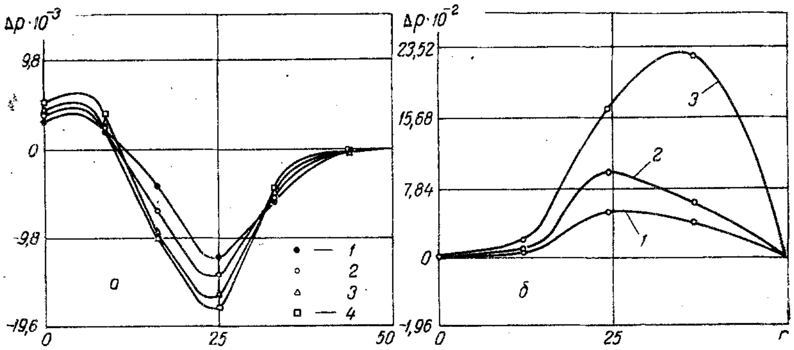

Fig. 2. Distribution of excess pressure along the radius of the interdisk gap: a) under normal atmospheric pressure in the ambeint medium $P_0 = 1.013 \cdot 10^5$ N/m² = 760 mm Hg and at n = 15,000 rpm: 1) h = 15 μ; 2) 10 μ; 3) 7.5 μ; b) under reduced ambient pressure $P_0 = 5.33 \cdot 10^3$ N/m² = 40 mm Hg and at n = 15,000 rpm: 1) 100 μ; 2) 80 μ; 3) 60 μ.

The Reiner effect can be analyzed by the Truesdell averaging method. If the transferable velocities vary within an infinitesimal physical volume, as has been assumed in the Predvoditelev theory, then the Navier–Stokes equation must contain average quantities designated here by brackets ⟨ ⟩.

In this case the Navier–Stokes equation becomes

$$\operatorname{div}(\langle \rho \mathbf{VV} \rangle) = -\nabla \langle P \rangle + \operatorname{div} \langle \bar{\bar{S}} \rangle + \langle \rho F \rangle + \frac{1}{V} \int_{\sigma} (\bar{\bar{S}} + \bar{\bar{\delta}} P)\, \mathbf{n}\, d\sigma, \tag{9}$$

where **n** is the unit vector normal to surface σ and $\bar{\bar{\delta}}$ is the unit tensor.

Thus, the modified Navier–Stokes equation (9) includes an additional term proportional to the force which is exerted by the gas at above the hydrostatic pressure and which is higher than any external force. If the disks are separated by a distance comparable to the free path of the molecules, therefore, then the quantities in the momentum transfer equation must be averaged. It is necessary to bear in mind here that, if l is the characteristic length within the volume over which velocity V has been averaged, then the condition

$$\lambda \ll l,$$

must prevail (λ denoting the free path length of the gas molecules).

The Reiner effect must be explained, therefore, beginning with the nonlinear relation between the stress tensor and the strain rate tensor, which is equivalent to introducing the Predvoditelev number or appropriately averaging the quantities in the momentum transfer equation. These additional terms may be considered to be parameters of the Reiner effect.

The test apparatus and the test procedure have been described in [7]. Small gaps between stator and rotor were measured with specially designed miniature inductive transducers. The sensitivity of the measuring circuit was 10-12 mV/μ. The tests were performed with a gap h ≥ 50 μ. In this case, according to formulas (3) and (4), perturbations due to design imperfections in the apparatus are reduced to a minimum.

The radial pressure distributions in the interdisk space are shown in Fig. 2 for various values of the gap with h, under normal and under reduced pressure. As can be seen here, the peak excess pressure shifts toward lower pressures of the ambient medium along the disk radius and it occurs at a distance (0.5-0.6) r from the disk axis.

NOTATION

$\dot{D}_{lm(0)}$ is the deviator of the strain rate tensor;
$S_{lm(0)}$ is the deviator of the stress tensor;
D is the deformation;
μ is the shear modulus;

η is the viscosity coefficient of the liquid;
τ is the time;
h is the gap;
ω is the angular velocity of the rotor;
P_0 is the ambient pressure;
ΔP is the excess pressure of the air;
$\mathbf{V}$ is the velocity;
η_V is the coefficient of bulk viscosity;
F is the external force;
ρ is the density;
V is the volume;
λ is the free path length;
r is the disk radius.

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